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No. 1.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from December Number.]

PROPOSITION XXXI. *Now I say there will be, of the aforesaid common perpendiculars in two distinct points, no determinate limit, such that under a smaller and smaller acute angle made at the point A, it would not always be possible to attain (in hypothesis of acute angle) to such a common perpendicular in two distinct points as is less than any assignable length R.*

PROOF. For in so far as the thing were otherwise; if from the point K (resume Fig. 30.) in BX assigned at any however great distance from the point B, a perpendicular KL is erected, to which from point A (by Euclid I. 12) the perpendicular AL is supposed let fall, KL ought to be greater than the length R.

The reason is; because a higher point Q being assumed in this BX, from which is erected to BX the perpendicular QF, to which (by the same Euclid I. 12) a perpendicular AF is let fall, this again must anyhow not be less than the length R.

But KL (from Corollary to preceding Proposition) will be greater than QF. Therefore KL would be greater than the aforesaid length R. And so ever proceeding higher.

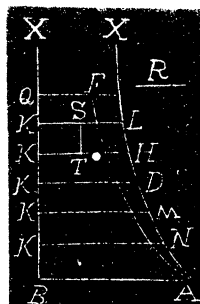


Fig. 30.

But now, if this however great KB is supposed divided (as in Proposition XXV) into portions KK , equal to the length R , and from these points K perpendiculars are erected, which meet AX in points H, D, M ; the angles at these points, toward the parts of the point L , will neither be right nor obtuse; lest in some quadrilateral, as suppose $KMLK$, the four angles together should be equal to or greater than four rights, contrary to the hypothesis of acute angle, according to which we are proceeding. Therefore all such angles will be acute toward the parts of the point L ; and therefore in like manner all at these points obtuse toward the parts of the point A . Wherefore (from Corollary I to Proposition III) of the aforesaid perpendiculars the least will indeed be KL more remote from the base AB , the greatest KM nearer this base.

And of the remaining the nearer will be ever greater than the more remote.

Therefore (from the preceding Proposition XXV, and its Corollary) the four angles together of the quadrilateral $KHLK$ more remote from base AB will be greater than the four angles together of all the remaining quadrilaterals nearer to this base. Wherefore (as in Proposition XXV) the hypothesis of acute angle would be destroyed.

Therefore it holds, that of the aforesaid common perpendiculars in two distinct points there will be no determinate limit, such that under a smaller and smaller acute angle made at the point A , it would not always be possible to attain (in hypothesis of acute angle) to such a common perpendicular in two distinct points as may be less than any assigned length R .

Quod erat demonstrandum.

[To be Continued.]

SOPHUS LIE'S TRANSFORMATION GROUPS.

A SERIES OF ELEMENTARY, EXPOSITORY ARTICLES.

By EDGAR ODELL LOVETT, Princeton, New Jersey.

IV.

PROOF OF LIE'S THEOREM THAT A ONE PARAMETER GROUP CONTAINS BUT ONE INFINITESIMAL TRANSFORMATION AND ITS CONVERSE THEOREM. EXAMPLES.

13. In the preceding paragraphs it has been shown by methods of proof due to LIE that every one parameter group with inverse transformations contains an infinitesimal transformation and conversely, every infinitesimal transformation generates a one parameter group. It is the purpose of this paragraph to present the proof of the theorem that the indefinite article "a" in these theorems can be replaced by the definite modifier "one and but one." The theorem